Boosts, radial ‘boosts’, and cosmic censorship

Nathan Carruth

SUSTech

3 September 2026

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Concentrated solutions to the Einstein vacuum equations in $U(1)$ symmetry: scaling, I

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- Specifically, we let γ denote the solution to the wave equation and (unimaginatively) write the metric as

$$h = \begin{pmatrix} 0 & 0 & -1 \\ 0 & (1+\delta\ell)^2 & b \\ -1 & b & c \end{pmatrix},$$

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  - Effectively, then, the self-coupling of γ to itself is *cubic*.
- After trying unsuccessfully to implement a *Gaussian beam* ansatz with this system, I (re)discovered the coordinate scaling

$$s \mapsto s,\ x \mapsto \delta^{-1/2}x,\ v \mapsto \delta^{-1}v$$

  and the scaling ansatz

$$\gamma(s, x, v) = \delta^{\iota}\overline{\gamma}(s, \delta^{-1/2}x, \delta^{-1}v),$$

  where $\overline{\gamma}$ is independent of δ to leading order.

Concentrated solutions in $U(1)$ symmetry: scaling, II

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  - We obtain existence up to $v \sim 1$, while Christodoulou/Klainerman-Rodnianski obtain existence only up to (effectively) $v \sim \delta^{-1}$.
  - On the other hand, our existence result requires that the initial data be Minkowskian to high order *after* a short incoming pulse, and this forces us to put *non-Minkowski* initial data on the *incoming* null hypersurface, unlike Christodoulou/Klainerman-Rodnianski.

Technical aside: Klainerman-Sobolev inequality on narrow strips

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- One obtains the expected $\sim r^{-1/2}$ decay by transforming to polar coordinates, but when attempting to complete the proof by applying the standard Sobolev inequality in polar coordinates one finds that one gains a factor $\sim r^{1/4}$ due to the *sharply peaked* nature of the region.

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- This gives an overall decay $\sim r^{-1/4}$.
- Given the cubic nature of the self-coupling of γ to itself, the error terms one obtains when estimating the wave γ are then essentially of size $\delta^{2\iota}(\delta^{-1}v)^{-1/2}$, meaning that we can only control γ up to $v \sim \delta^{1-4\iota}$, hence the requirement $\iota \geq 1/4$.

Concentrated solutions without symmetry: scaling

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- Naïvely, we could consider initial data (say in terms of initial incoming shear, to follow the literature)

$$\hat{\chi}(u, \underline{u}, x, y) \sim \delta^{\iota - 1} \overline{\hat{\chi}}(u, \delta^{-1}\underline{u}, \delta^{-1/2}x, \delta^{-1/2}y).$$

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- Now it is not hard to see that if one takes data of the above size and performs the scaling transformation

$$u \mapsto u,\ \underline{u} \mapsto \delta^{-1}\underline{u},\ x \mapsto \delta^{-1/2}x, y \mapsto \delta^{-1/2}y,$$

then one ought to obtain data which is a sufficiently small perturbation of *Minkowski* data that any standard local existence result, e.g., the classical result of Rendall, will give existence over some small region of size $O(1)$ *in the scaled picture.*

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- A moment's thought dispels this notion, since (physically speaking) the above solution is simply a *Lorentz boosted* unconcentrated beam.
- It is easy to see that one ought to be able to construct spacetimes with two *colliding* beams of the above form, which might give something more interesting.

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  - The first is to note that one need not solve on the entire sphere at once; one can solve on a large number of patches of size $\sim \delta^{1/2}$.
  - The second, more important, is to note that if we replace the usual polar coordinate θ by $\theta - \pi/2$, so that the round sphere metric becomes $d\theta^2 + \cos^2\theta d\phi^2$, then applying the above scaling, together with an overall conformal scaling by δ^{-1} as before, the Minkowski metric is of the form

$$\eta = -2dud\underline{u} + [r_0 + 2^{-1/2}(\delta\underline{u} - u)]^2(d\theta^2 + \cos^2(\delta^{1/2}\theta)d\phi^2).$$

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  - It thus makes sense to use η_∞ as the 'base metric' in an application of Rendall's existence results.

Radial ‘boosts’ giving trapped surface formation: overall idea

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- The foregoing reduces Christodoulou’s original 500+ page monograph to perhaps a dozen pages.
- Much more importantly, it provides a far more versatile tool for obtaining trapped surface formation in other settings.

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- I am very interested in working on other techniques for approaching cosmic censorship questions.